Failure Analysis of FRP Composite Laminates Using Progressive Failure Criteria

RENJIN J BRIGHT, SUMATHI M

Abstract— The prime factors to be considered while designing a composite material are the stresses acting on the composite lamina and laminate. This could be determined by means of lamination theory and the failure stress region could be predicted by means of composite failure criteria. There are numerous failure criteria to predict failure of Fibre Reinforced Polymer (FRP) composite materials. However, traditionally used failure criteria such as the Tsai-Hill and Tsai-Wu failure criteria developed in the 1960s and 70s continues to rule the industries. Nevertheless, the fluctuating properties of composite materials in each direction demands much more developed failure criteria which could precisely predict the fibre failure and matrix failure respectively. Development of new failure criteria entails deep insight of the existing failure criteria, which is a tedious task. This work provides a brief outlook about the later developed and modified failure criteria such as Hashin-Rotem, Hashin, Rotem, Edge, Sun and Puck which could predict failure accurately. The failure predictability of these failure criteria has been assessed by comparing with the commonly used failure criteria and experimental data by means of generating failure envelopes, considering the case of a wind turbine blade. A software has been developed for easy admittance of the above mentioned failure criteria and to perform failure analysis of composite laminate with different ply orientation. The reliability of the newly developed software has been validated by performing failure analysis of a balanced and symmetric quasi-isotropic composite laminate (0°/ ± 45°/90°)S with the aid of empirical relations and by means of the finite element analysis package ANSYS.

Index Terms —Lamina, Laminate, Lamination Theory, Failure Stress, Failure Criteria, Failure Envelope, Fibre Failure, Matrix failure.

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1. INTRODUCTION

The mechanisms for complete laminate failure are best understood by initialstudy of lamina failure. Initial microscopic failures of composites can be represented by failure modes, such as delamination, matrix tensile, matrix compressive, fibre tensile and fibre compressive failure modes. For industrial practices failure theories have to be validated with experimental evidence. The experimental validation and compendium of some of the accurate failure theories have been executed by a resolute exercise named as worldwide failure exercise (WWFE) [1,2]. Assessment of failure and failure theories of composite materials are being carried out assiduously. A critical review of important failure theories has been done and the major deficiencies were quoted by [3]. The critical fracture plane concept which is responsible for fibre failure, matrix failure and ply by ply failure of composite materials has been discussed in [4]. [5] discussed about various composite failure theories and approaches for failure prediction. The stress-based Grant-Sanders method developed at British Aerospace Defence, which was applied to number of examples to produce failure envelopes of initial and final failures has been explained in [6]. A comprehensive study of lamina and laminate failure criteria has been done and a fracture plane concept was developed for failure in unidirectional composites under various loading cases by [7]. The fracture plane concept developed by [7] relating to the fracture at fibre matrix interface has beenrefined by [8].

In this work ten failure criteria including the four best ranked criteria such as Edge [6], Sun [7], Puck [8], Rotem [9], and commonly used composite failure criteria such as Tsai-Hill [10], Tsai-Wu [11], Hashin [12], Hashin-Rotem [13], Maximum Stress [14], Maximum Strain [14], have been demonstrated.

2. OVERVIEW OF FAILURE CRITERIA

Failure criteria for composite materials are classified into two groups namely non-interactive failure criteria and interactive Failure Criteria [14]. A non-interactive failure criterion is the one having no interactions between stress or strain components. An interactive failure criterion is the one having interaction between stress or strain components. Table.1 demonstrates the compendium of maximum exploited failure criteria compiled from various literatures. The nomenclature utilized in the construction of failure criteria is illustrated in table.2. Among these, Maximum stress and strain criteria falls under non-interactive failure criteria and all other theories falls under interactive failure criteria.

Conferring to maximum stress criteria, failure occurs when at least one stress component (σ_{11}, σ_{22}, τ_{12}) along the principal material axes exceeds the corresponding ultimate strength in that direction, while according to maximum strain criteria failure occurs when at least one of the strain components (ε_{11}, ε_{22}, γ_{12}) along the principal material axes exceeds corresponding ultimate strain in that direction [14]. Tsai-Hill failure criteria explains failure based on Von-Mises' distortional energy yield criterion [10], while Tsai and Wu constructed a failure criteria assuming the existence of failure surface in stress space and in- plane shear strength similarity

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[11].

TABLE 1

Sl. No	***Criterion***	***Equation***

COMPENDIUM OF FAILURE CRITERIA OF FRP COMPOSITES

1.	Maximum Stress Criteria	$\sigma_{11} = \begin{Bmatrix} X_T \text{when } \sigma_{11} \geq 0 \\ -X_C \text{when } \sigma_{11} \leq 0 \end{Bmatrix}; \sigma_{22} = \begin{Bmatrix} Y_T \text{when } \sigma_{22} \geq 0 \\ Y_C \text{when } \sigma_{22} \leq 0 \end{Bmatrix}; \tau_{12} = S_{LT}$
2.	Maximum Strain Criteria	$\varepsilon_{11} = \begin{Bmatrix} X_T when \varepsilon_{11} \geq 0 \\ -X_C when \varepsilon_{11} \leq 0 \end{Bmatrix}; \varepsilon_{22} = \begin{Bmatrix} Y_T when \varepsilon_{22} \geq 0 \\ Y_C when \varepsilon_{22} \leq 0 \end{Bmatrix}; \gamma_{12} = S_{LT}$
3.	Tsai- Hill Criteria	$\frac{{\sigma_{11}}^2}{{F_1}^2} + \frac{{\sigma_{22}}^2}{{F_2}^2} + \frac{{\tau_{12}}^2}{{F_6}^2} - \frac{\sigma_{11}\sigma_{22}}{{F_1}^2} = 1$ *where,* $F_1 = \begin{Bmatrix} X_T when \sigma_{11} \geq 0 \\ X_C when \sigma_{11} \leq 0 \end{Bmatrix}; F_2 = \begin{Bmatrix} Y_T when \sigma_{22} \geq 0 \\ Y_C when \sigma_{22} \leq 0 \end{Bmatrix}$ and $F_6 = S_{LT}$
4.	Tsai-Wu Criteria	$H_1\sigma_{11} + H_2\sigma_{22} + H_{11}{\sigma_{11}}^2 + H_{22}{\sigma_{22}}^2 + H_{66}{\tau_{12}}^2 + 2H_{12}\sigma_{11}\sigma_{22} = 1$ where, $H_1 = \frac{1}{X_T} - \frac{1}{X_C}; H_{11} = \frac{1}{X_T X_C}; H_2 = \frac{1}{Y_T} - \frac{1}{Y_C}; H_{22} = \frac{1}{Y_T Y_C};$ $H_{22} = \frac{1}{{S_{LT}}^2}; H_{12} = -[H_{11}H_{22}]^{1/2}$
5.	Hashin-Rotem Criteria	Fiber Failure : $\left(\frac{\sigma_{11}}{\pm X_T}\right)^2 + \left(\frac{\tau_{12}}{S_{LT}}\right)^2 = 1$ Matrix Failure : $\left(\frac{\sigma_{22}}{\pm Y_T}\right)^2 + \left(\frac{\tau_{12}}{S_{LT}}\right)^2 = 1$
6.	Hashin's Criteria	Fiber Failure : $\left(\frac{\sigma_{11}}{X_T}\right)^2 + \left(\frac{\tau_{12}}{S_{LT}}\right)^2 = 1$; Fiber Compression : $\left(\frac{\sigma_{11}}{X_C}\right) = 1$ Matrix Tension : $\left(\frac{\sigma_{22}}{Y_T}\right)^2 + \left(\frac{\tau_{12}}{S_{LT}}\right)^2 = 1$ Matrix Compression : $\left(\frac{\sigma_{22}}{2S_{TT}}\right)^2 + \left[\left(\frac{Y_C}{2S_{TT}}\right)^2 - 1\right]\left(\frac{\sigma_{22}}{Y_C}\right) + \left(\frac{\tau_{12}}{S_{LT}}\right)^2 = 1$
7.	Rotem's Criteria	Fiber Failure: $\sigma_{11} = \begin{Bmatrix} X_T \text{when } \sigma_{11} \geq 0 \\ -X_C \text{when } \sigma_{11} \leq 0 \end{Bmatrix}; \sigma_{22} = \begin{Bmatrix} Y_T \text{when } \sigma_{22} \geq 0 \\ Y_C \text{when } \sigma_{22} \leq 0 \end{Bmatrix};$ $\tau_{12} = S_{LT}$ Matrix Tension: $\left(\frac{E_m \varepsilon_{11}}{Y_{mT}}\right)^2 \left(\frac{\sigma_{22}}{Y_T}\right)^2 + \left(\frac{\tau_{12}}{S_{LT}}\right)^2 = 1$ Matrix Compression : $\left(\frac{E_m \varepsilon_{11}}{Y_{mC}}\right)^2 \left(\frac{\sigma_{22}}{Y_C}\right)^2 + \left(\frac{\tau_{12}}{S_{LT}}\right)^2 = 1$
8.	Edge's Criteria	Initial failure: Matrix Tension: $\sigma_{22} = Y_T$; Matrix Compression: $\sigma_{22} = Y_C$ Combined Shear and Matrix Tension: $\left(\frac{\sigma_{22}}{Y_C}\right)^2 + \left(\frac{\tau_{12}}{S_{LT}}\right)^2$ For final failure: Fiber Tension: $\sigma_{11} = X_T$; Fiber Compression: $\sigma_{11} = X_C$ Combined Shear and Fiber Compression: $\left(\frac{\sigma_{11}}{X_T}\right)^2 + \left(\frac{\tau_{12}}{S_{LT}}\right)^2 = 1$
9.	Sun's Criteria	Fiber Tension $\frac{\sigma_{11}}{X_T} = 1$; Fiber Compression: $\frac{\sigma_{11}}{X_C} = 1$ Matrix Tension: $\left(\frac{\sigma_{22}}{Y_T}\right)^2 + \left(\frac{\tau_{12}}{S_{LT} - \mu\sigma_{22}}\right) = 1$ Matrix Compression: $\left(\frac{\sigma_{22}}{Y_C}\right)^2 + \left(\frac{\tau_{12}}{S_{LT}}\right)^2 = 1$
10.	Puck's Criteria	Fiber Tension : $\frac{1}{\varepsilon_{1T}}\left(\varepsilon_{11} + \frac{\gamma_{f12}}{E_{f1}} m_{\sigma_f}\sigma_{22}\right) = 1$ Fiber Compression: $\frac{1}{\varepsilon_{1C}}\left(\varepsilon_{11} + \frac{\gamma_{f12}}{E_{f1}} m_{\sigma_f}\sigma_{22}\right) = 1 - (10\gamma_{21})^2$ IFF Mode A $(\sigma_{22} \geq 0)$: $\sqrt{\left(\frac{\tau_{12}}{S_{LT}}\right)^2 + \left(1 - p_1^+ \frac{\tau_{12}}{S_{LT}}\right)^2 \left(\frac{\sigma_{22}}{Y_T}\right)^2} + p_1^+ \frac{\sigma_{22}}{S_{LT}} = 1 - \left\lvert\frac{\sigma_{11}}{\sigma_{11D}}\right\rvert$ IFF Mode B $\left(\sigma_{22} < 0 \quad \& \; 0 \leq \frac{\sigma_{22}}{\tau_{12}} \leq \frac{R_{11}^A}{\tau_{12C}}\right): \frac{1}{S_{LT}}\left(\sqrt{(\tau_{12})^2 + (p_1^- \sigma_{22})^2} + p_1^- \frac{\sigma_{22}}{S_{LT}}\right) = 1 - \left\lvert\frac{\sigma_{11}}{\sigma_{11D}}\right\rvert$ IFF Mode C $\left(\sigma_{22} < 0 \quad \& \; 0 \leq \frac{\tau_{12}}{\sigma_{22}} \leq \frac{\tau_{12C}}{R_{11}^A}\right): \left(\frac{1}{2(1 + p_{11}^-)}\right)\left(\left(\frac{\tau_{12}}{S_{LT}}\right)^2 + \left(\frac{\sigma_{22}}{R_{11}^A}\right)^2\right)\left(\frac{R_{11}^A}{-\sigma_{22}}\right)$ $= 1 - \left\lvert\frac{\sigma_{11}}{\sigma_{11D}}\right\rvert$ $R_{11}^A = \left(\frac{S_{LT}}{2p_1^-}\right)\left(\sqrt{1 + 2p_1^- \frac{Y_c}{S_{LT}}} - 1\right); p_{11}^- = R_{11}^A \frac{p_1^-}{S_{LT}}$

TABLE 2
NOMENCLATURE

Variable	*Description*	*Unit*
E_{11}	Longitudinal Modulus	GPa
E_{22}	Transverse Modulus	GPa
G_{12}	Shear Modulus	GPa
υ_{12}	Major Poisson's ratio	-
υ_{21}	Minor Poisson's ratio	-
E_m	Matrix Modulus	GPa
σ_{11}, σ_{22}, τ_{12}	Stresses in Longitudinal, Transverse and shear Direction respectively	MPa
ε_{11}, ε_{22}, ε_{12}	Strains in Longitudinal, Transverse and shear Direction respectively	-
X_T, X_C	Lamina Tensile and compressive Longitudinal Strengths respectively	MPa
Y_T, Y_C	Lamina Tensile and compressive Transverse Strengths respectively	MPa
S_{LT}, S_{TT}	Lamina in-plane and Transverse Shear Strengths respectively	MPa
Y_{mT}, Y_{mC}	Matrix Transverse and Compressive Strengths	MPa
ε_c^T	Normal Strain in Transverse Direction	-
ε_c^L	Normal Strain in Longitudinal Direction	-
μ	Sun's Internal Material Friction Parameter	-
θ	Fracture angle	Degrees
σ_n, τ_{nt}, τ_{n1}	Fracture Plane Stresses	MPa
R_{11}^A	Fracture resistance of the action plane against its fracture due to transverse/transverse shear stress	MPa
p_1^+, p_1^-	Slopes of Puck's (σ_{11}-τ_{12}) Fracture Envelopes	-
p_{11}^-	Slope of Puck's (σ_n-τ_{nt}) Fracture Envelopes at (σ_n=0)	-

Hashin and Rotem used their experimental observations on tensile specimens to propose two different failure criteria, one related to fiber failure and the other related to matrix failure [13]. Hashin introduced fibre and matrix failure criteria that distinguish between tension and compression failure [12]. Rotem explained the fibre failure and matrix failure separately by considering matrix tension and matrix compression [9]. E.C Edge differentiated composite failure into initial and final failures where initial failure explained failure modes of matrix in tension, compression and combined tension and compression, while final failure demonstrated the same failure modes of fibre [6]. Sun proposed an empirical modification to Hashin's criterion. This criterionconsiders maximum stress criteria for fibre failure and formulated new one for matrix failure which includes a coefficient named as internal friction parameter (μ), considering the resistance to failure offered by compressive stress [7]. Puck's criteria have been evolved from extensive experimental studies of the mechanisms by which failure occurs in a lamina when subjected to a biaxial stress state [8]. Puck's criteria introduced several ply cracking mechanisms and consideration of the orientation angle of the fracture plane were also made as shown in figure 1.

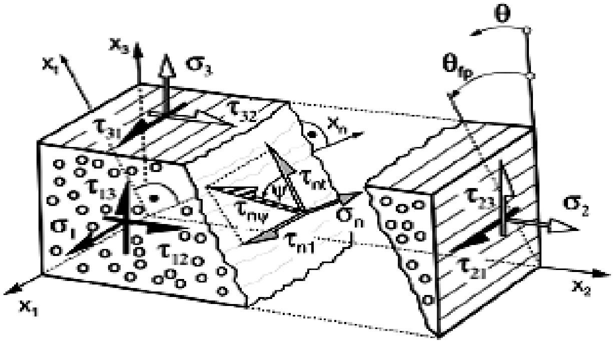


Figure 1. Stresses τ_{nt}, τ_{nl}, and σ_n on Fibre Parallel Plane [8]

3. COMPARATIVE STUDY OF FAILURE CRITERIA

Figure 2 to 4 depicts the failure predictability of the lamina failure criteria discussed in section 2. For better comparison of the failure criteria the lamina failure envelopes have been generated considering the application of a wind turbine blade. The materials and experimental data employed for the comparative study were taken from [15]. The materials for plotting failure envelopes have been selected based on the availability of test data, in order to compare the level of conservatism of each failure criteria. The properties of materials and respective test data used for the failure envelope generation is shown in table 3 and table 4 respectively.

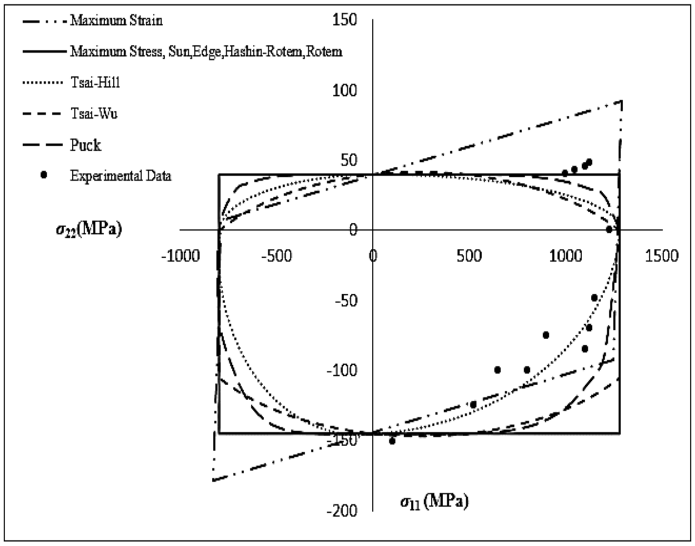


Figure 2. Comparison of Lamina Failure Criteria in 'σ_{11}-σ_{22}' bi-axial stress field for material 1

TABLE 3

PROPERTIES OF COMPOSITE MATERIALS FOR WIND TURBINE BLADE [15]

Properties	*Composite Material*		
Fiber Matrix	E-glass LY556/HT907/DY063 Epoxy (Material-1)	E-Glass MY750/HY917/DY063 Epoxy (Material-2)	T300 BSL914C Epoxy (Material-3)
E_f (GPa)	80	74	230
υ_f	0.2	0.2	0.2
E_m (GPa)	3.35	3.35	4
υ_m	0.35	0.35	0.35
E_{11} (GPa)	53.48	45.6	138
E_{22} (GPa)	17.7	16.2	11
G_{12} (GPa)	5.83	5.83	5.5
$\upsilon 12$	0.278	0.278	0.28
X_T (MPa)	218	1280	1500
X_C (MPa)	176	800	900
Y_T (MPa)	36	40	27
Y_C (MPa)	138	145	200
S_{LT} (MPa)	61	73	80

TABLE 4

EXPERIMENTAL DATA OF COMPOSITE MATERIALS CONSIDERED [15]

Material-1		*Material-2*		*Material-3*	
σ_{11}	σ_{22}	σ_{11}	τ_{12}	σ_{22}	τ_{12}
1000	40	1500	0	40	0
1050	43	1450	50	26.9	36
1100	46	1320	72	30.7	32.3
1125	48	1300	70	34	12.8
1230	0	1000	125	18	51.3
1150	-48	800	120	-137.8	0
1125	-70	775	130	-142	0
1100	-85	750	110	-132.3	0
900	-75	0	100	-134.6	46.7
800	-100	-100	95	-123	28.9
650	-100	-250	90	-99	64.5
525	-125	-400	80	-70.5	96.6
100	-150	-600	75	-122	54.6
		-700	60	-44	81.9
		-750	46	-133	20.7
		-800	30	0	61.2
		-900	0		

3.1. Comparison of Lamina Failure Criteria in 'σ_{11}-σ_{22}' Biaxial Stress Field

Figure 2 depicts the failure predictability of lamina failure criteria in 'σ_{11}-σ_{22}' stress field (both longitudinal and compressive). All criteria fit well with the experimental data in the first quadrant. Among them puck and maximum stress criteria have been found to be well in agreement. In the fourth quadrant the failure prediction of Puck, Tsai-Wu and Tsai-Hill criteria fits very much to the experimental data. Lack of experimental data limits the failure prediction in second and third quadrant.

3.2. Comparison of Lamina Failure Criteria in 'σ_{11}-τ_{12}' Biaxial Stress Field

Figure 3 depicts the failure predictability of lamina failure criteria in 'σ_{11}-τ_{12}' bi-axial stress field. Only positive region of shear stress has been considered since the value of shear stress is same in both region and thus the failure envelope is symmetric above x-axis. From the failure envelope it is clear that none of the criteria considered here predicts the failure. In tensile region of longitudinal stress none of the selected criteria fits with the experimental data while puck's theory predicts the failure to some extent.

3.3. Comparison of Lamina Failure Criteria in 'σ_{22}-τ_{12}' Biaxial Stress Field

Figure 4 depicts the failure predictability of lamina failure criteria in 'σ_{22}-τ_{12}' bi-axial stress field. During operation, leading edges of the wind turbine blade are subjected to compressive loading. Field survey of failed blades proved that the induced in-plane shear stresses are the main cause of blade failure. As a result, 'σ_{22}-τ_{12}'stress failure (Matrix Failure) is given most importance than any other mode.

In Tensile region, all predictions corresponding to each selected Criteria are similar. Also they fit well with experimental data. It also shows that in tensile mode, with increase in tensile stress there will be a corresponding decrease in shear stress. This implies that tensile load prompts failure either with presence or absence of shear stress. The most interesting behaviour develops when 'σ_{22}' becomes compressive. The experimental data shows an increase of shear strength as 'σ_{22}' attains compression failure mode. Hashin-Rotem (1973) criterion gives an elliptical envelope with reducing 'τ_{12}' value as compressive 'σ_{22}' increases. The envelope for Hashin's criteria (1980) was calculated using a transverse strength and it reflects an improvement in accuracy compared to the 1973 criterion. Edge follows Maximum stress theory in the tensile region and Hashin's criteria at the compressive region. Tsai-Wu criteria also predict the failure on the compressive region but it does not discriminate the failure mode. Among all the criteria shown in figure 4, Sun's and Puck's criteria fits well with the experimental data. Puck's failure envelope is very much accurate, but it relies on fitting parameters. The failure envelope for Sun's criterion was calculated using 'μ', which is explained to be the internal friction parameter, but it does not have any theoretical support. The results of Sun's criteria indicate a significant improvement over Hashin's criteria.

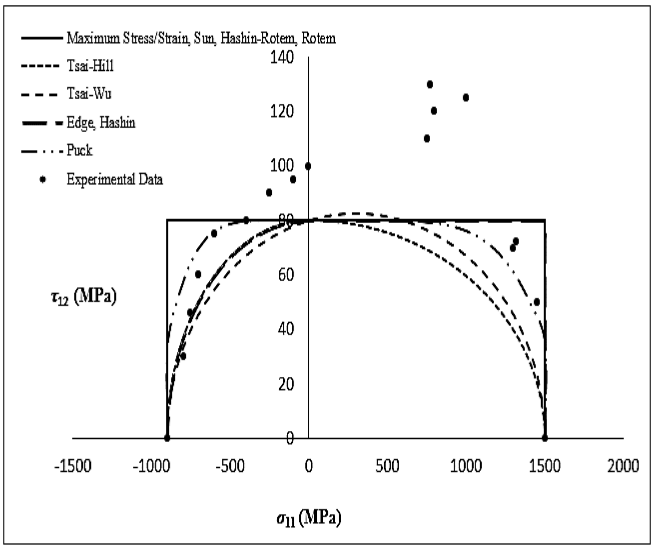


Figure 3. Comparison of Lamina Failure Criteria in 'σ_{11}-τ_{12}' bi-axial stress field for material 2

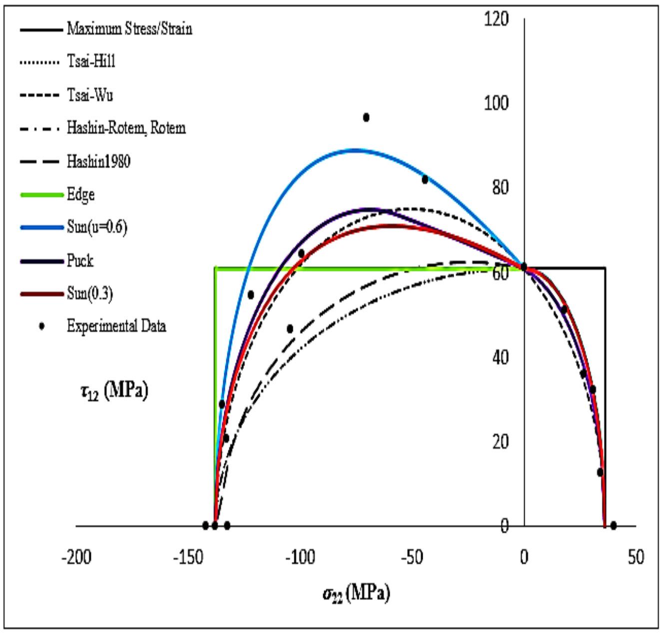


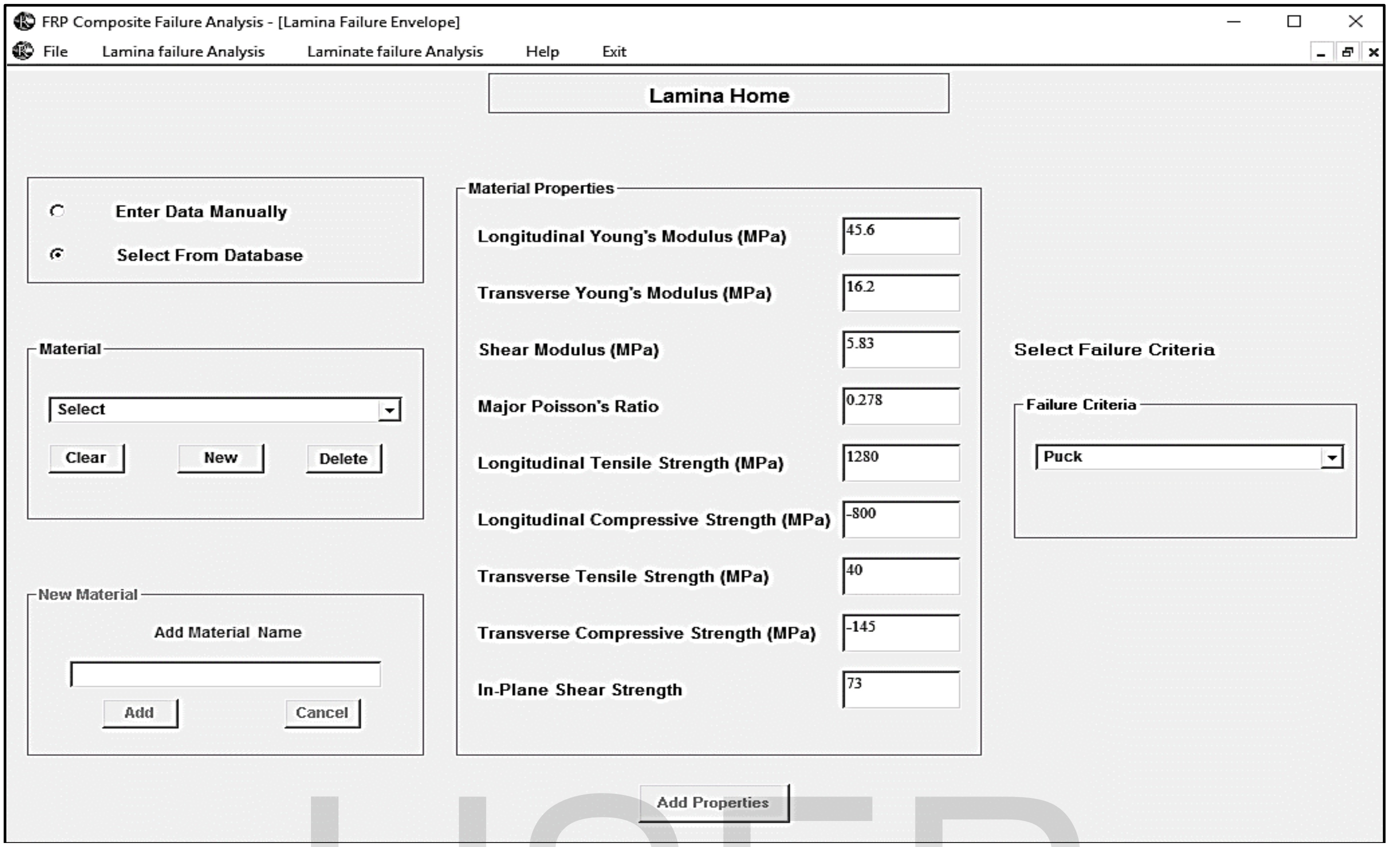
Figure 4. Comparison of Lamina Failure Criteria in 'σ_{22}-τ_{12}' bi-axial stress field for material 3

4. DEVELOPMENT OF SOFTWARE FOR FRP COMPOSITE FAILURE ANALYSIS

In this work a new software has been developed in order to perform the failure analysis of FRP composites from lamina level to laminate level in a less complicated manner. The software has been developed using Visual Basic-6 as front end and MS Access as back end. The software is comprised of two modules specifically, lamina failure envelope generation module as depicted in figure 5 and laminate failure analysis module as depicted in figure 6. In lamina failure envelope module, failure envelopes could be generated for the failure criteria discussed above and could be compared with each other and with that of the experimental data. The comparison of failure envelopes of above mentioned failure criteria for 'σ_{11}-σ_{22}', 'σ_{11}-τ_{12}' and 'σ_{22}-τ_{12}' stress fields generated using the newly developed software is depicted in figures 7-9. In laminate failure analysis module, laminate design parameters such as laminate stresses, strains and failure indices of each constituent lamina could be found out. From figures 7 to 9 it could be concluded that lamina failure envelopes generated using the software and by empirical means are same.

The laminate failure analysis module has been utilized to evaluate [A], [B], [D] matrices, laminate stresses and failure indices of a quasi-isotropic composite laminate (0°/ ± 45°/90°)S. The results have been compared with that obtained by empirical means and by finite element analysis.

The database of this software has been provided with some commonly used composite materials and is availed with the facility of adding new materials into it. All the failure criteria have been explained in a brief and user friendly manner along the procedure to carry out lamina and laminate failure analysis in a separate module named as ‘Help’.



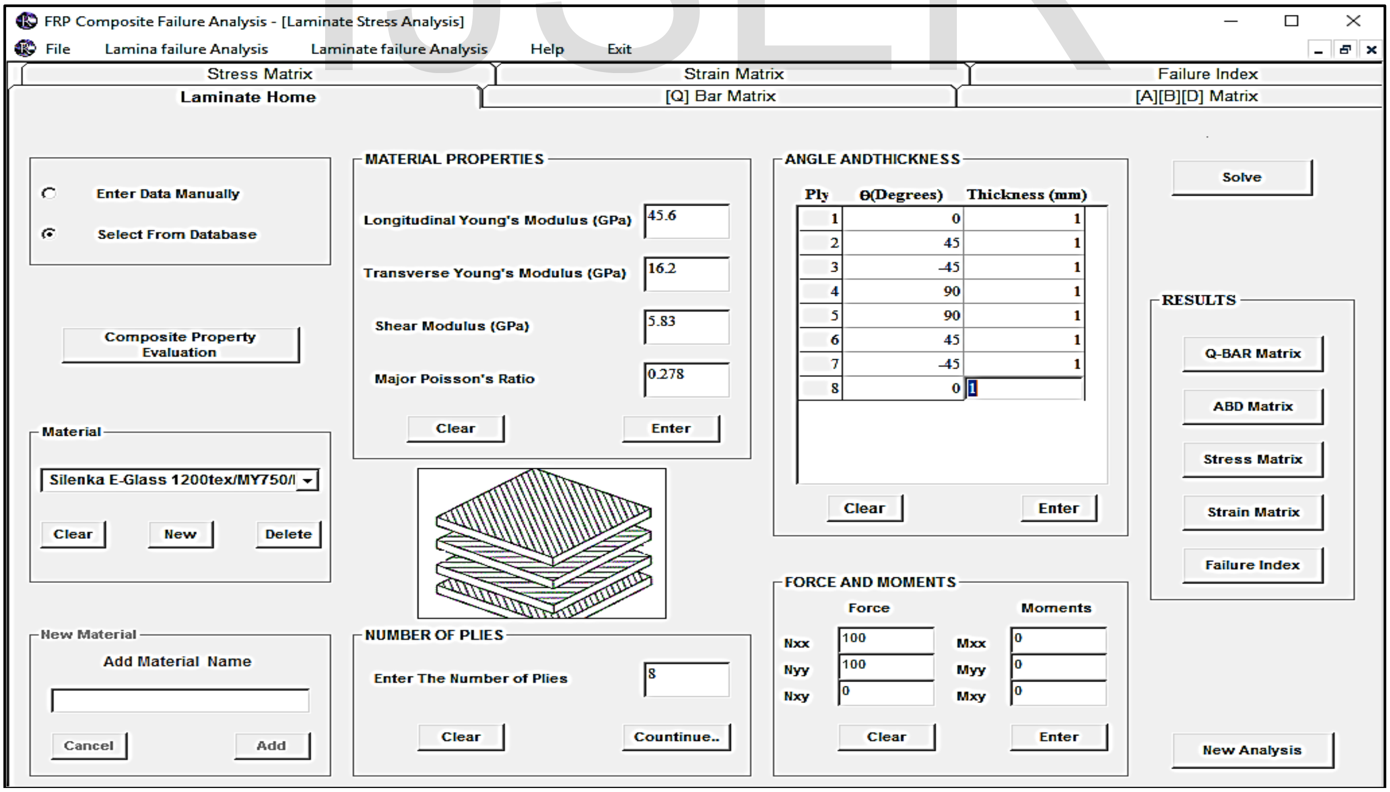
○ Enter Data Manually
● Select From Database

Material: Select

Failure Criteria: Puck

Longitudinal Young's Modulus (MPa)	45.6
Transverse Young's Modulus (MPa)	16.2
Shear Modulus (MPa)	5.83
Major Poisson's Ratio	0.278
Longitudinal Tensile Strength (MPa)	1280
Longitudinal Compressive Strength (MPa)	-800
Transverse Tensile Strength (MPa)	40
Transverse Compressive Strength (MPa)	-145
In-Plane Shear Strength	73

Figure 5. Lamina Failure Analysis Module



○ Enter Data Manually
● Select From Database

Material: Silenka E-Glass 1200tex/MY750/I

Longitudinal Young's Modulus (GPa)	45.6
Transverse Young's Modulus (GPa)	16.2
Shear Modulus (GPa)	5.83
Major Poisson's Ratio	0.278

Enter The Number of Plies: 8

Ply	θ(Degrees)	Thickness (mm)
1	0	1
2	45	1
3	-45	1
4	90	1
5	90	1
6	45	1
7	-45	1
8	0	1

Force		Moments	
Nxx	100	Mxx	0
Nyy	100	Myy	0
Nxy	0	Mxy	0

Figure 6. Laminate Failure Analysis Module

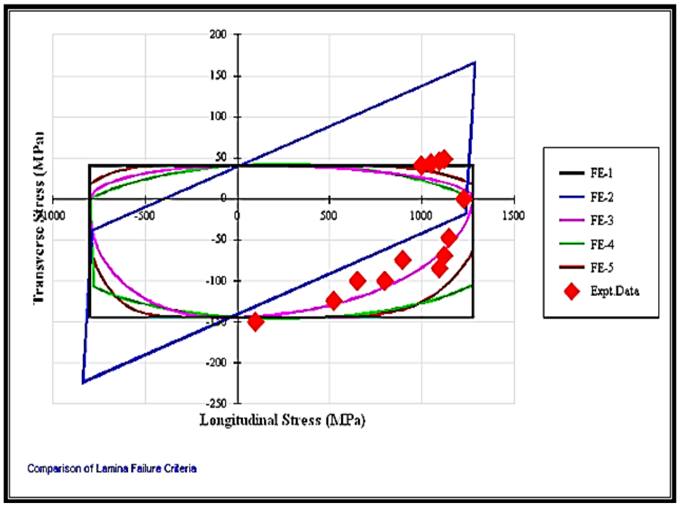


Figure 7. Comparison of Lamina Failure Criteria in 'σ_{11}-σ_{22}' bi-axial stress field for material 1 using the newly developed software

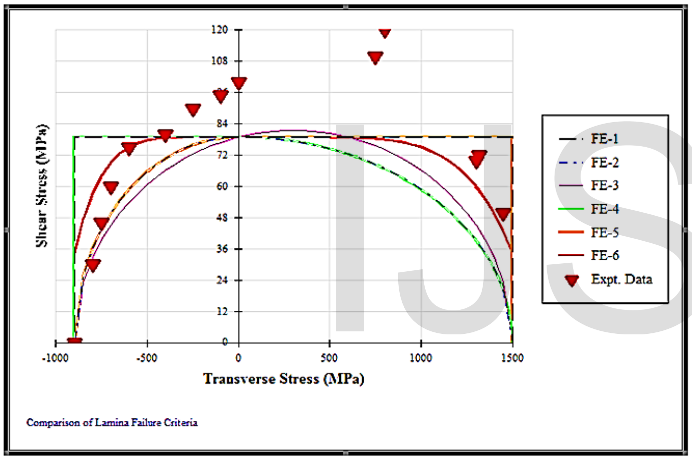


Figure 8. Comparison of Lamina Failure Criteria in 'σ_{11}-τ_{12}' bi-axial stress field for material 2 using the newly developed software

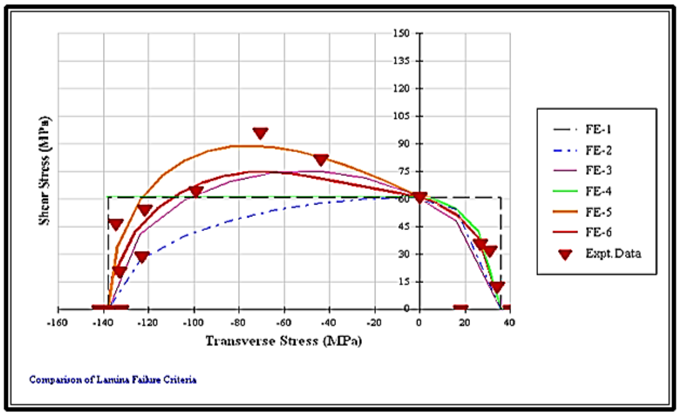


Figure 9. Comparison of Lamina Failure Criteria in 'σ_{22}-τ_{12}' bi-axial stress field for material 3 using the newly developed software

Figure 7 explains the comparison of lamina failure criteria in 'σ_{11}-σ_{22}' biaxial stress field generated by means of the newly developed software. When comparing figure 7 with figure 2, it could be observed that the software developed provides similar results as that obtained by theoretical method. In order to reduce the complexity in the plot area of graph, the legends were made available in a new window activated by means of button click. Here legends FE-1 states Maximum Stress criteria, Hashin's criteria, Rotem's criteria, sun's criteria and Edge's failure criteria, FE-2 states Maximum Strain failure criteria, FE-3 states Tsai-hill failure criteria, FE-4 states Tsai-Wu failure criteria and FE-5 states Puck's failure criteria.

Figure 8 explains the comparison of lamina failure criteria in 'σ_{11}-τ_{12}' biaxial stress field generated by means of the software developed. When comparing figure 8 with figure 3, it could be observed that the software developed provides similar results as that obtained by theoretical method. Here legends FE-1states Maximum Stress criteria, Rotem's criteria, Maximum Strain criteria and sun's failure criteria, FE-2 states Tsai-hill failure criteria, FE-3 states Tsai-Wu failure criteria, FE-4 states Hashin's failure criteria, FE-5 states Edge's failure criteria and FE-6states puck's failure criteria.

Similarlyfigure 9 explains the comparison of lamina failure criteria in 'σ_{22}-τ_{12}' biaxial stress field generated by means of the software developed. When comparing figure 9 with figure 4, it could be observed that the software developed provides similar results as that by theoretical method. Here legends FE-1states maximum stress criteria, and maximum strain failure criteria, FE-2 states Tsai-hill criteria, Rotem's criteria and Hashin's failure criteria, FE-3 states Tsai-Wu failure criteria, FE-4 states Edge's failure criteria, FE-5 states Sun's failure criteria and FE6-states puck's failure criteria.

5. FINITE ELEMENT ANALYSIS OF FRP COMPOSITE LAMINATE

In this section an attempt has been made to analyse the failure of FRP composite laminate by means of Finite Element Analysis using the software ANSYS. For this work the balanced and symmetric (0°/ ± 45°/90°) quasi-isotropic laminate (each layer of thickness 1mm), made of T300-BSL914C-Epoxy concerning the application of a wind turbine blade has been considered whose orientation is shown in figure 10. Two dimensional model of the composite specimen is created using the element type 'linear shell 181' and results have been generated for different loading conditions. The results obtained such as [A], [B] and [D] matrices are shown in figure 7. Laminate stresses and failure indices have been compared to that obtained by theoretical means and by the newly developed software. Table. 5 compares [A], [B] and [D] matrices obtained from ANSYS, by theoretical method and by the newly developed software.

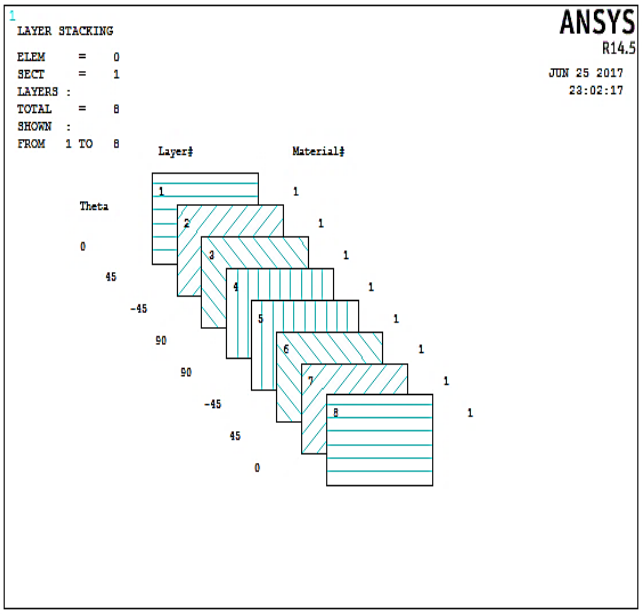
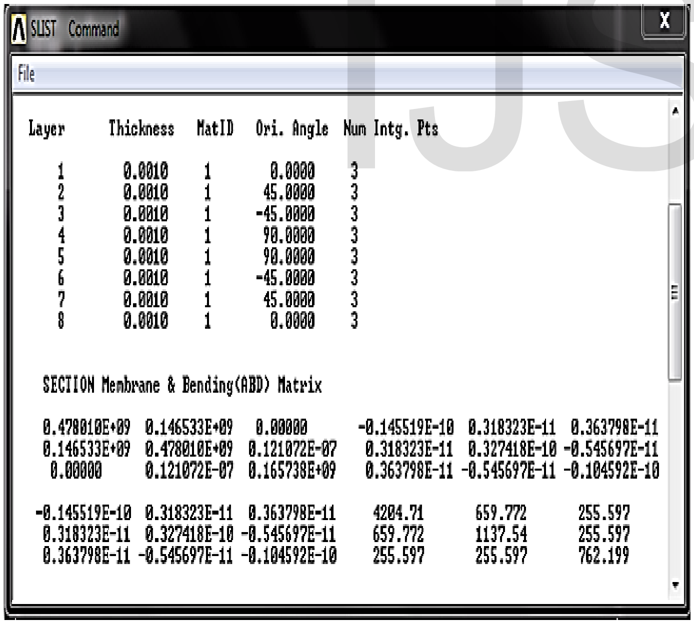


Figure 10. Layer orientation of (0°/ ± 45°/90°)S quasi-isotropic laminate

5.1. Evaluation of Laminate [A], [B] and [D] matrices

The [A], [B] and [D] matrices obtained from ANSYS as shown in figure 7.



```
Layer      Thickness   MatID   Ori. Angle  Num Intg. Pts

   1        0.0010      1        0.0000      3
   2        0.0010      1       45.0000      3
   3        0.0010      1      -45.0000      3
   4        0.0010      1       90.0000      3
   5        0.0010      1       90.0000      3
   6        0.0010      1      -45.0000      3
   7        0.0010      1       45.0000      3
   8        0.0010      1        0.0000      3


 SECTION Membrane & Bending(ABD) Matrix

 0.478010E+09  0.146533E+09  0.00000       -0.145519E-10  0.318323E-11  0.363798E-11
 0.146533E+09  0.478010E+09  0.121072E-07   0.318323E-11  0.327418E-10 -0.545697E-11
  0.00000      0.121072E-07  0.165738E+09   0.363798E-11 -0.545697E-11 -0.104592E-10

-0.145519E-10  0.318323E-11  0.363798E-11    4204.71       659.772       255.597
 0.318323E-11  0.327418E-10 -0.545697E-11    659.772       1137.54       255.597
 0.363798E-11 -0.545697E-11 -0.104592E-10    255.597       255.597       762.199
```

Figure 11. [A], [B], [D] matrices of (0°/ ± 45°/90°)s quasi-isotropic laminate

The concept of [A], [B] and [D] matrices and the empirical relationship for determining these matrices are explained in equations (1-4). Figure 12 depicts the co-ordinate locations of ply in a laminate [14]. From table 5, it could be concluded that [A], [B] and [D] matrices obtained from ANSYS, by theoretical means and by the new software developed are similar.

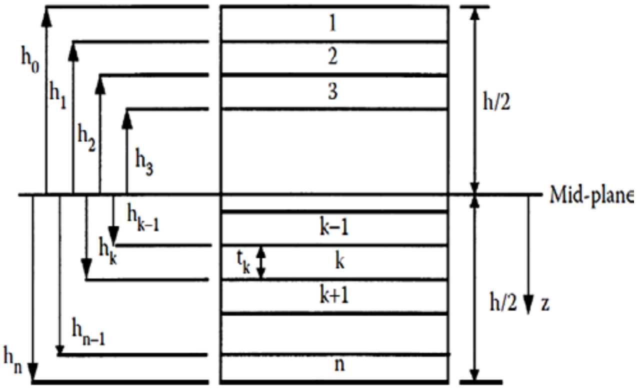


Figure 12. Co-Ordinate Locations of Ply in Laminate[14]

The [A], [B], [D] Matrices can be determined by[14],

$$A_{ij} = \sum_{k=1}^{n} [\bar{Q}_{ij}]_k (h_k - h_{k-1}) \quad (1)$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} [\bar{Q}_{ij}]_k ({h^2}_k - {h^2}_{k-1}) \quad (2)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} [\bar{Q}_{ij}]_k ({h^3}_k - {h^3}_{k-1}) \quad (3)$$

where

[A] = extensional stiffness matrix for the laminate in Pa-m
[B] = coupling stiffness matrix for the laminate in Pa-m^2
[D] = bending stiffness matrix for the laminate in in Pa-m^3
h = Lamina Thickness in mm
n = Number of lamina

$[\bar{Q}_{ij}]$ = Transformation reduced stiffness Matrix in GPa. It is given by,

$$[\bar{Q}_{ij}] = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66} \end{bmatrix} \quad (4)$$

5.2. Evaluation of Laminate Stresses and Failure Indices

Here a balanced and symmetric quasi-isotropic laminate (0°/ ± 45°/90°)S is analysed for failure by determining the resulting stresses and failure indices of each lamina. The computation is done by means of FEA using the software ANSYS for a load of 100 kN. The longitudinal, transverse and shear stresses of each lamina could be evaluated using ANSYS. Table 6 to 8 depicts the comparison between the stresses in each layer of the laminate obtained from ANSYS, by theoretical method and by the software developed for different loading conditions such as longitudinal, transverse and combined longitudinal and transverse loading. The comparison discussed in table 6 explains that the stresses of each layer of the laminate obtained from ANSYS, by theoretical method and by the software developed is of negligible deviation. Table 7 figures out the failure index values for the laminate obtained from ANSYS, by theoretical method and by the newly developed software. Failure criteria such as Maximum stress and Tsai-wu criteria used in ANSYS is utilized for evaluating failure indices.

TABLE 5.

COMPARISON OF [A], [B] AND [D] MATRICES OBTAINED BY VARIOUS METHODS

	ANSYS	*Software Developed*	*Theoretical*
[A]	$\begin{bmatrix} 47.801 & 14.653 & 0 \\ 14.653 & 47.81 & 1.2E^{-15} \\ 0 & 1.2E^{-15} & 16.57 \end{bmatrix}$	$\begin{bmatrix} 47.801 & 14.653 & -0.0001 \\ 14.653 & 47.81 & -0.0051 \\ 0 & 0 & 16.57 \end{bmatrix}$	$\begin{bmatrix} 47.801 & 14.653 & -0.0001 \\ 14.653 & 47.81 & -0.0051 \\ 0 & 0 & 16.57 \end{bmatrix}$
[B]	$\begin{bmatrix} -1.45 & 0.31 & 0.36 \\ 0.31 & 3.2 & -0.54 \\ 0.36 & -0.54 & -1.04 \end{bmatrix} \times 10^{-11}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
[D]	$\begin{bmatrix} 4204.71 & 659.77 & 255.59 \\ 659.77 & 1137.54 & 255.59 \\ 255.59 & 255.59 & 762.19 \end{bmatrix}$	$\begin{bmatrix} 4204.71 & 659.77 & 255.59 \\ 659.77 & 1137.54 & 255.59 \\ 255.59 & 255.59 & 762.19 \end{bmatrix}$	$\begin{bmatrix} 4204.71 & 659.77 & 255.59 \\ 659.77 & 1137.54 & 255.59 \\ 255.59 & 255.59 & 762.19 \end{bmatrix}$

TABLE 6

STRESS ON LAMINATE SUBJECTED TO COMBINED LONGITUDINAL AND TRANSVERSE LOAD OF 100 KN AND 50 KN RESPECTIVELY, CONSIDERING SYMMETRY

	ANSYS (MPa)			*Software Developed (MPa)*			*Theoretical (MPa)*		
	σ_{11}	σ_{22}	τ_{12}	σ_{11}	σ_{22}	τ_{12}	σ_{11}	σ_{22}	τ_{12}
0°	-9.11	2.34	-0.533×10^{-16}	-9.113	2.34	0	-9.11	2.34	0
45°	4.59	7.91	5.12	4.59	7.91	5.12	4.59	7.91	5.12
-45°	4.59	7.91	-5.12	4.59	7.91	-5.12	4.59	7.91	-5.12
90°	-0.067	31.8	0.17×10^{-14}	-0.067	31.84	0	-0.067	31.84	0

TABLE 7

FAILURE INDICES OF EACH LAYER OF THE LAMINATE SUBJECTED TO COMBINED LONGITUDINAL AND TRANSVERSE LOAD OF 100 KNAND 50 KN, CONSIDERING SYMMETRY

Layer		*0°*	*45°*	*-45°*	*90°*
ANSYS	Max. Stress	0.040755	0.063017	0.063017	0.0852
	Tsai-Wu	0.023549	0.047455	0.047455	0.0715
Software Developed	Max. Stress	0.408	0.063	0.063	0.0853
	Tsai-Wu	0.0232	0.0471	0.0471	0.0714
Theoretical	Max. Stress	0.408	0.063	0.063	0.0853
	Tsai-Wu	0.0232	0.0471	0.0471	0.0714

The comparison discussed in table 7 explains that the failure indices of each layer of the laminate obtained from ANSYS, by theoretical method and by the software developed is of negligible deviation.

5.3. LAMINATE FIRST PLY FAILURE (FPF) LOAD

First ply failure load of a laminate could be determined by means of the failure indices evaluated as discussed above. For determining first ply failure load, the maximum failure index value among the lamina were selected and is utilized accordingly with respect to the failure criteria used. As the failure indices values were different for the Maximum Stress and Tsai-Wu criteria the first ply failure load would also be of different values. First ply failure load is determined by equation (5). For the investigation of first ply failure load failure indices have been evaluated for longitudinal and transverse loading separately.

$$\text{FPF} = \text{Applied Load} \times \frac{1}{\text{F. I}_{\text{max}}} \quad (5)$$

F. I $_{\text{max}}$ denotes maximum failure index. First Ply failure load using Maximum Stress and Tsai-Wu failure criteria is given in table 7.

TABLE 7

FIRST PLY FAILURE LOAD OF THE (0°/ ± 45°/90°)S LAMINATE (KN)

FPF	*Maximum Stress Criterion*	*Tsai-Wu Criterion*	*Corresponding Lamina*
Longitudinal Load	1156.069	1282.051	90°
Transverse Load	1156.069	1219.51	0°

6. CONCLUSION

In this work a comparative study of ten selected failure criteria has been done and their failure predictability has been evaluated by means of generating failure envelopes. For failure envelope generation of composite lamina, a new software has been developed. The reliability of the newly developed software has been accessed by empirical means and by FEA. An attempt has also been made to evaluate laminate stresses and failure indices of each lamina using FEA. The results obtained were compared with that of the newly developed and that obtained by empirical means. All the three results were found to be similar with negligible deviation. Effective utilization of failure indices for computing first ply failure load has been demonstrated. First ply failure load evaluation can be extended for the evaluation of the last ply failure load by performing progressive analysis. Future scope of this work deals with the development of a new module in the software developed which could be utilized for conducting progressive laminate failure. It is also premeditated to modify the most effective Puck's failure criteria in order to reduce its conservatism and predict failure accurately.

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